

## VJEŽBA 9.

### Centrično pritisnut stub kvadratnog poprečnog presjeka

#### Zadatak 1.

Armiranobetonski stub, slobodno oslonjen na obje strane, visine  $h=3.0$  m, kvadratnog poprečnog presjeka, centrično je pritisnut silom

$$N_g = 1000 \text{ kN} \quad N_p = 1200 \text{ kN}$$

- Dimenzionisati stub ne uzimajući u obzir vitkost (kratak stub);
- Zadržavajući dimenzije stuba iz tačke a) odrediti potrebnu armaturu za slučajeve  $\lambda=60$  i  $\lambda=90$ .

Kvalitet usvojenog materijala: MB 30 RA 400/500  
Nacrtati plan armature poprečnog presjeka stuba u razmjeri R 1:10.

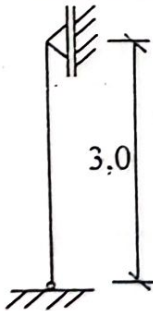
#### Rješenje:

- Kratak stub (izvijanje se ne uzima u obzir)

##### a.1. Ulazni podaci

$$\begin{aligned} N_g &= 1000 \text{ kN} & N_p &= 1200 \text{ kN} \\ \text{MB 30} & \Rightarrow & f_b &= 20.5 \text{ MPa} \\ \text{RA 400/500} & \Rightarrow & \sigma_v &= 400 \text{ MPa} \end{aligned}$$

##### a.2. Statički sistem



##### a.3. Statički uticaji

$$N_u = 1,9 \times N_g + 2,1 \times N_p = 1,9 \times 1000 + 2,1 \times 1200 = 4420 \text{ kN}$$

##### a.4. Dimenzionisanje

$$\text{Usvojeno } \min \mu = 0,6\% \quad \bar{\mu} = \mu \alpha \frac{\sigma_v}{f_b} = 0,6 \times \frac{400}{20,5} = 11,71\%$$

$$A_b = \frac{N_u}{f_b \times (1 + \bar{\mu})} = \frac{4420}{2,05 \times \left(1 + \frac{11,71}{100}\right)} = 1930,1 \text{ cm}^2$$

$$b = \sqrt{A_b} = 43,9 \text{ cm}$$

Usvojeno:  $b=45 \text{ cm}$

### a.5. Kontrola vitkosti

$$h=l=3.0 \text{ m}$$

$$i = \frac{b}{\sqrt{2}} = \frac{45}{\sqrt{2}} = 13 \text{ cm}$$

$$\lambda = \frac{h}{i} = \frac{300}{13} = 23,1 < 25$$

Ne vrši se provjera stabilnosti na uticaje izvijanja  
(član 105 PBAB '87 knjiga 2)

### a.6. Određivanje armature

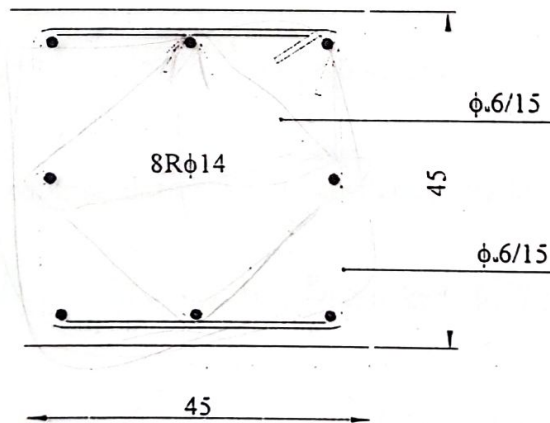
$$\bar{\mu} = \frac{N_u}{f_b \times A_b} - 1 = \frac{4420}{2.05 \times 45^2} - 1 = 0.0647 = 6.47\%$$

$$\mu = \bar{\mu} \times \frac{\sigma_v}{f_b} = 6.47 \times \frac{2.05}{40} = 0.33\%$$

$$\min \mu = 0.3 \times \left( 1 + \frac{\sigma_b}{f_b} \right) = 0.3 \times \left( 1 + \frac{4420}{45 \times 45 \times 2.05} \right) = 0.619\% > 0.33\%$$

$$A_a = \frac{0.619}{100} \times 45 \times 45 = 12.54 \text{ cm}^2$$

Usvaja se : 8R $\phi$ 14 (12.32 cm<sup>2</sup>)



### b) Vitki stubovi

#### b.1. Umjereno vitak stub ( $25 \leq \lambda_r \leq 75$ )

$$\lambda = 60 \text{ i } h = 7.8 \text{ m}$$

$$e = e_0 + e_1 + e_\varphi + e_2$$

$e$  – ukupna ekcentričnost

$e_0$  – ekcentričnost usljed udstupanja od vertikale pri izvođenju

$$e_1 - e_1 = \frac{M}{N}$$

$e_\varphi$  – ekcentričnost usljed uticaja tečenja

$e_2$  – ekcentričnost koju izaziva sila pritiska usled deformacija stuba, uticaji drugog reda

b.1.2. Metoda dopunske ekcentričnosti za  $25 \leq \lambda_i \leq 75$ , važi za  $\lambda=60$

$e_0$   $e_0 = \frac{h_i}{300} = \frac{780}{300} = 2,6 \text{ cm}$ , treba da važi  $2 \text{ cm} \leq e_0 \leq 10 \text{ cm}$

$e_1$   $M=0$ , pa je  $e_1=0$

$e_\varphi$  Efekti tečenja mogu biti zanemareni ako je ispunjen jedan od sljedećih uslova

$\lambda_i \leq 50$	$\lambda_i = 60$	$\left. \begin{array}{l} \text{nije} \\ \text{nije} \\ \text{nije} \end{array} \right\} \text{ tečenje se ne može zanemariti}$
$e_1/d > 2$	$e_1/d = 0$	
$N_g \leq 0.2N_g$	$1000 > 0.2 \times (1000 + 1200) = 440$	

$N_g = \text{statiko}$   
 $N_g = \text{ukupna sila}$

$$e_\varphi = (e_{1g} + e_0) \times \left( e^{\frac{\alpha_E}{1-\alpha_E} \varphi} - 1 \right), \quad \alpha_E = \frac{N_g}{N_E}$$

$$N_E = \frac{E_b I_b \pi^2}{h_i^2} \text{ Ojlerova sila izvijanja}$$

$$N_E = \frac{3150 \times \frac{45^4}{12} \times \pi^2}{780^2} = 17462 \text{ kN}; \quad N_g = 1000 \text{ kN}; \quad \alpha_E = \frac{1000}{17462} = 0,057$$

Za MB 30  $E_b = 31500 \text{ MPa}$  PBAB '87, član 52, tabela 8.  
 $\varphi_\infty = 2,5$  PBAB '87, član 59, tabela 11.

$$e_\varphi = 2,6 \times \left( e^{\frac{0,057}{1-0,057} \times 2,5} - 1 \right) = 0,42 \text{ cm}$$

$e_2$  Postupak dopunske ekcentričnosti za  $\lambda_i \leq 75$

$$\frac{e_2}{d} = \frac{\lambda_i - 25}{100} \times \sqrt{0,10 + \frac{e_1}{d}} \geq 0 \text{ kada je } 0 \leq \frac{e_1}{d} \leq 0,30$$

$$\frac{e_2}{d} = \frac{\lambda_i - 25}{160} \geq 0 \text{ kada je } 0,3 \leq \frac{e_1}{d} < 2,5$$

$$\frac{e_2}{d} = \frac{\lambda_i - 25}{160} \times \left( 3,5 - \frac{e_1}{d} \right) \text{ kada je } 2,5 \leq \frac{e_1}{d} < 3,5$$

$$e_1 = 0, \text{ pa je } e_2 = \frac{\lambda_i - 25}{100} \times \sqrt{0,10} \times d = \frac{60 - 25}{100} \times \sqrt{0,10} \times 45 = 4,98$$

$$e = e_0 + e_1 + e_\varphi + e_2 = 2,6 + 0 + 0,42 + 4,98 = 8 \text{ cm}$$

b.1.3. Statički uticaji

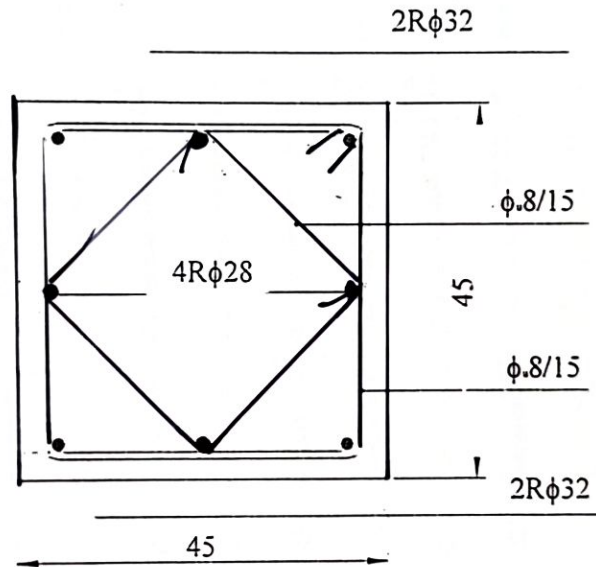
$$M_u = 0,08 \times 4420 = 353,6 \text{ kNm} \text{ i } N_u = 4420 \text{ kN}$$

#### b.1.4. Dimenzionisanje

$$\left. \begin{aligned} m_u &= \frac{353.6 \times 100}{45^3 \times 2.05} = 0,189 \\ n_u &= \frac{4420}{45^2 \times 2.05} = 1,065 \end{aligned} \right\} \bar{\mu}_1 = \bar{\mu}_2 = 0,26$$

$$A_{a1} = A_{a2} = \frac{0,26 \times 45 \times 45}{2,05} = 26,98 \text{ cm}^2, \text{ te je } A_a = 53,98 \text{ cm}^2$$

Usvaja se: 4R $\phi$ 28+4R $\phi$ 32 (56,8 cm<sup>2</sup>)



#### b.2. Vitak stub $\lambda_i > 75$

$$\lambda_i = 90 \text{ i } h_i = 11,7 \text{ m}$$

##### b.2.1. Određivanje minimalnog procenta armiranja

$$\min \mu = \frac{\lambda_i}{50} - 0,4 = \frac{90}{50} - 0,4 = 1,4\% \quad \text{član 188. PBAB'87}$$

$$\min \bar{\mu} = \min \mu \times \frac{\sigma_v}{f_b} = 1,4 \times \frac{40}{2,05} = 27,32\%$$

##### b.2.2. Određivanje ekcentriciteta

$$e_0 = \frac{l_i}{300} = \frac{1170}{300} = 309 \text{ cm, imperfekcija}$$

$$e_1 = 0, \text{ početni ekcentricitet } M=0$$

$$e_\varphi = (e_{1g} + e_0) \times \left( e^{\frac{\alpha_E \varphi}{1 - \alpha_E}} - 1 \right), \quad \alpha_E = \frac{N_g}{N_E}$$

$$N_E = \frac{3150 \times \frac{45^4}{12} \times \pi^2}{1170^2} = 7760,8 \text{ kN}; \quad N_g = 1000 \text{ kN}; \quad \alpha_E = \frac{1000}{7760,8} = 0,129$$



$$\bar{K} = F_m \cdot S_{Igv} / b \cdot d^3 \cdot f_b$$

		.00	.10	.15	.20	.25	.30	.35	.40	.45	.50	.60	.70	.80	.90	1.00	1,10	1.20	
n	k	$m \cdot 10^3 = 10^3 \cdot \mu_u / b \cdot d^3 \cdot f_b$																	
1.10	.5	0	0	13	19	24	28	32	35	39	42	48	54	60	65	71	76	81	
	1.0	0	0	18	34	45	54	62	69	76	82	95	107	119	130	141	151	162	
	1.5	0	0	0	37	53	69	84	99	110	120	140	158	176	192	209	225	241	
	2.0	0	0	0	0	56	73	90	107	123	140	172	204	229	251	272	293	312	
	2.5	0	0	0	0	0	0	94	111	129	146	181	214	248	280	313	345	374	
	3.0	0	0	0	0	0	0	0	133	151	166	186	221	255	289	323	356	390	
	3.5	0	0	0	0	0	0	0	0	0	0	0	0	260	295	330	364	399	
	max k	0	0	1.2	1.7	2.1	2.4	2.6	2.8	3.0	3.2	3.3	3.4	3.6	3.7	3.8	3.9	4.0	
	max m	0	0	19	38	56	75	94	113	133	152	188	225	261	297	333	369	406	
1.20	.5	0	0	0	0	17	22	27	31	35	38	45	51	57	63	69	74	79	
	1.0	0	0	0	0	0	36	51	60	68	75	89	101	113	125	136	147	158	
	1.5	0	0	0	0	0	0	56	72	88	104	130	149	167	185	202	219	235	
	2.0	0	0	0	0	0	0	0	76	93	110	144	177	210	243	265	287	308	
	2.5	0	0	0	0	0	0	0	0	0	0	114	149	185	219	254	287	320	
	3.0	0	0	0	0	0	0	0	0	0	0	0	189	225	260	295	329	364	
	3.5	0	0	0	0	0	0	0	0	0	0	0	0	0	300	336	371	406	
	max k	0	0	0	0	1.0	1.5	1.8	2.1	2.4	2.6	3.0	3.2	3.3	3.5	3.6	3.7	3.8	
	max m	0	0	0	0	19	38	57	76	95	114	153	191	228	264	301	337	374	
1.30	.5	0	0	0	0	0	0	0	0	18	25	30	34	41	48	54	60	66	
	1.0	0	0	0	0	0	0	0	0	38	54	66	74	81	95	108	120	132	
	1.5	0	0	0	0	0	0	0	0	0	57	74	90	107	125	143	161	179	
	2.0	0	0	0	0	0	0	0	0	0	0	0	113	147	181	215	248	279	
	2.5	0	0	0	0	0	0	0	0	0	0	0	0	152	188	223	258	293	
	3.0	0	0	0	0	0	0	0	0	0	0	0	0	0	192	228	264	300	
	3.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	341	
	max k	0	0	0	0	0	0	0	0	1.8	1.3	1.7	1.9	2.4	2.8	3.1	3.2	3.3	
	max m	0	0	0	0	0	0	19	39	58	77	115	154	192	230	267	304	341	
1.40	.5	0	0	0	0	0	0	0	0	0	0	19	29	37	45	51	58	64	
	1.0	0	0	0	0	0	0	0	0	0	0	38	58	72	88	102	115	127	
	1.5	0	0	0	0	0	0	0	0	0	0	0	76	110	143	169	188	205	
	2.0	0	0	0	0	0	0	0	0	0	0	0	0	115	150	185	219	253	
	2.5	0	0	0	0	0	0	0	0	0	0	0	0	0	154	190	226	262	
	3.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	231	268	304	
	3.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	341	
	max k	0	0	0	0	0	0	0	0	0	0	.7	1.1	1.8	2.2	2.6	2.9	3.1	
	max m	0	0	0	0	0	0	0	0	20	39	77	116	154	193	232	270	307	
1.50	.5	0	0	0	0	0	0	0	0	0	0	0	0	33	41	48	55	61	
	1.0	0	0	0	0	0	0	0	0	0	0	0	0	39	74	95	109	121	
	1.5	0	0	0	0	0	0	0	0	0	0	0	0	0	77	113	146	179	
	2.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	116	152	188	
	2.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	192	229	
	3.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	270	
	3.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	341	
	max k	0	0	0	0	0	0	0	0	0	0	0	0	1.0	1.6	2.1	2.4	2.7	
	max m	0	0	0	0	0	0	0	0	0	0	39	78	116	155	194	233	271	

$$e_{\varphi} = 3,9 \times \left( 2,718^{\frac{0,129}{1-0,129} \times 2,5} - 1 \right) = 1,75 \text{ cm}$$

Ukupan bezdimenzionalni ekcentricitet I reda je:

$$e_1 = e_0 + e_1 + e_{\varphi} = 3,9 + 1,75 = 5,65 \text{ cm}$$

$$\frac{e_1}{d} = \frac{5,65}{45} = 0,126$$

### b.2.3. Dimenzionisanje

Usvojeno: simetrično armiran stub

$$a/h = 0,1, \quad h = 45 - 0,1 \times 45 = 40,5 \text{ cm}$$

$$n = \frac{N_u}{b x d x f_b} = \frac{4420}{45 \times 45 \times 2,05} = 1,06 \text{ usvojeno } n_r = 1,1 > n = 1,06$$

#### 1. Aproksimacija

$$k = \varepsilon \times h \times 10^3 = 2 \quad \Leftrightarrow \quad \varepsilon = \frac{2,0 \times 10^3}{40,5} = 4,938 \times 10^{-5} \text{ 1/cm}$$

- ekcentricitet

$$e_2 = 0,1 \times \varepsilon \times h_i^2 = 0,1 \times 4,928 \times 10^{-5} \times 11,7^2 \times 10^4 = 6,76 \text{ cm}$$

$$e/d = e_1/d + e_2/d = 0,126 + 6,76/45 = 0,276 \text{ cm}$$

- potreban moment unutrašnjih sila

$$m_{\text{potr}} = n_r \times \frac{e}{d} = 1,1 \times 0,276 = 0,304$$

- potrebna armatura  $\bar{\mu}$  ( $m_r \times 10^3$ ) = 1,1(293) - 1,2(312)

$$m_{\text{potr}} \bar{\mu}_r = 1,1 + \frac{1,2 - 1,1}{312 - 293} \times (304 - 293) = 1,158 > 0,273 = \min \bar{\mu}$$

#### 2. Aproksimacija

$$k = 2,5 \quad \Leftrightarrow \quad \varepsilon = \frac{2,5 \times 10^3}{40,5} = 6,17 \times 10^{-5} \text{ 1/cm}$$

- ekcentricitet

$$e_2 = 0,1 \times 6,17 \times 10^{-5} \times 11,7^2 \times 10^4 = 8,45 \text{ cm}$$

$$e/d = e_1/d + e_2/d = 0,126 + 8,45/45 = 0,314 \text{ cm}$$

- potreban moment unutrašnjih sila

$$m_{\text{potr}} = n_r \times \frac{e}{d} = 1,1 \times 0,314 = 0,345$$

- potrebna armatura  $\bar{\mu}$  ( $m_r \times 10^3$ ) za  $m_r = 0,345$

$$m_{\text{potr}} \bar{\mu}_r = 1,1\%$$

#### 3. Aproksimacija

$$k = 3,0 \quad \Leftrightarrow \quad \varepsilon = \frac{3,0 \times 10^3}{40,5} = 7,407 \times 10^{-5} \text{ 1/cm}$$

- ekcentricitet

$$e_2 = 0,1 \times 7,407 \times 10^{-5} \times 11,7^2 \times 10^4 = 10,139 \text{ cm}$$

$$e/d = e_1/d + e_2/d = 0.126 + 10.139/45 = 0.351 \text{ cm}$$

- potreban moment unutrašnjih sila

$$m_{tr} = n_r \times \frac{e}{d} = 1.1 \times 0.351 = 0.386$$

- potrebna armatura  $\bar{\mu} (m_r \cdot 10^3) = 1.1(356) - 1.2(390)$

$$m_{tr} \bar{\mu} = 1.1 + \frac{1.2 - 1.1}{390 - 356} \times (386 - 356) = 1.19$$

Iz aproksimacija 1, 2 i 3 tražimo  $\min \bar{\mu}_r \implies \bar{\mu}_r = 1.1\%$

$$A_a = \bar{\mu}_r \times b \times d \times \frac{f_b}{\sigma_v} = 1.1 \times 45^2 \times \frac{2.05}{40} = 114.16 \text{ cm}^2$$

Usvaja se: 12R $\phi$ 36 (122,16 cm<sup>2</sup>)

